# "Cheat Sheet" for Algebra
# Math 101, Littlefield

**Order of operations for standard notation: "PEMDAS"**

Repeat as needed, at every level:

1. work separately above and below any fraction bar
2. evaluate anything inside grouping operators first — **P**arentheses (), square brackets [], absolute value ||, expressions inside an exponent $a^{like+here}$ or inside a radical $\sqrt[n]{like+here}$
3. do **E**xponentiation ($a^n$) and radicals ($\sqrt[n]{a}$)
4. do **M**ultiplication and **D**ivision, from left to right
5. do **A**ddition and **S**ubtraction, from left to right

**When in doubt, parenthesize everything and explicitly indicate all multiplications.**

**You can always substitute equals for equals.**

For example, if $a=b+c$ and $ad=e$, then $(b+c)d=e$.
But remember, <u>the parentheses are important.</u>
If you leave them out, then $a=b+c$ and $ad=e$ turns into $b+cd=e$, and that just is <u>not</u> right!

**If you do the same thing to both <u>entire</u> sides of equality, you still have equality.**

For example, **<u>IF</u>** $a=b$, then:

$(a) + c = (b) + c$ for any $c$ [the parentheses are to emphasize **<u>entire</u>** sides]
$(a) – c = (b) – c$ for any $c$
$(a) * c = (b) * c$ for any $c$
$(a) / c = (b) / c$ for any $c$, except undefined if $c=0$
$(a)^n = (b)^n$ for any n, except undefined if $a <= 0$ and $n < 0$

Beware of spurious solutions if you multiply or divide by something that might be zero, or whenever you raise both sides to an even power (including zero).

**Handy operations that preserve equality** (When in doubt, test with some real numbers!)

For any $a$, $b$, $c$:

If $a = b$ and $b = c$, then $a = c$ [equality is *transitive*]

$(a+b)+c = a+(b+c)$ [addition is *associative*]
[but subtraction is **<u>not</u>** associative: $(a–b) –c \neq a– (b–c)$ ]
(a*b)*c = a*(b*c) [multiplication is a*ssociative*]
[but division is **<u>not</u>** associative: (a/b)/c ≠ a/(b/c) ]

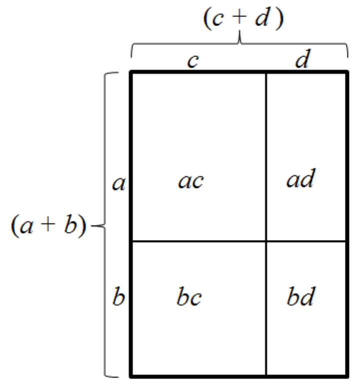
$a+b = b+a$ [addition is *commutative*]
[but subtraction is **<u>not</u>** commutative: a–b ≠ b–a ]
a*b = b*a [multiplication is *commutative*]
[but division is **<u>not</u>** commutative: a/b ≠ b/a ]

$a*(b+c) = a*b + a*c$ [multiplication distributes across addition]
[but division does **<u>not</u>** distribute across addition: $a/(b+c) \neq a/b + a/c$ ]

$(a+b)*(c+d) = a*c + a*d + b*c + b*d$ [more generally, all pairs. Some books say FOIL: First, Outer, Inner, Last]

| | |
|---|---|
| $a+0 = 0+a = a$ | [0 is the *additive identity*] |
| $a+(-a) = 0$ | [$-a$ is the *additive inverse* of $a$] |
| $a+(-b) = a-b$ | [handy rules for negatives] |
| $a-(-b) = a+b$ | |
| $a*(-b) = -a*b$ | |
| $(-a)*(-b) = a*b$ | [product of two negatives is a positive] |
| $a*1 = 1*a = a$ | [1 is the *multiplicative identity*] |
| $a/a = 1$ | |
| $a*(1/a) = 1$ | [1/a is the *multiplicative inverse* of a] |
| $a*0 = 0*a = 0$ | [the *multiplication property of 0*] |

"Area model of multiplication"




$(a+b)\cdot(c+d) = ac + ad + bc + bd$
FOIL: *First Outer Inner Last*

**Expanding:**

$a*(c+d) \rightarrow a*c + a*d$

**Factoring:**

$a*c + a*d \rightarrow a*(c+d)$

**Cancelling common factors:**

$(a*c)/(b*c) \rightarrow a/b$

**Fractions:**

Adding and subtracting

$a/b + c/d = (ad+bc)/(bd)$ [convert to common denominator bd, then add or subtract the numerators]

$a/b - c/d = (ad-bc)/(bd)$

Multiplying

$(a/b) * (c/d) = (ac)/(bd)$

Dividing

$(a/b) / (c/d) = (a/b) * (d/c)$ [invert and multiply]

Distributing addition

$(a+b)/c = (a/c) + (b/c)$

Reducing (cancelling)

$\frac{(c)\cdot(a)}{(c)\cdot(b)} = \frac{a}{b}$ [see the separate handout for details!]

**Handy rule for solving equations of the form *a*b* = 0:**

If $a*b = 0$, then either $a=0$, or $b=0$, or both.

So, for example,
if $(x-s)*(x-t) = 0$, then either $(x-s)=0$ [and thus $x=s$], or $(x-t)=0$ [and thus $x=t$].