"Cheat Sheet" #2 for Algebra: Powers, Roots, and Logarithms
Math 101, Littlefield

Powers

$a^0 = 1$ for any a

$a^N = \overbrace{a \cdot a \cdots a \cdot a}^{N\ copies}$ for any positive integer N

$a^{\frac{1}{N}} = \sqrt[N]{a}$ for any positive integer N (Read as: *N'th root of a*)

$a^{\frac{K}{N}} = \sqrt[N]{a^K} = \left(\sqrt[N]{a}\right)^K$ (Read as: *N'th root of a to the power K*)

$a^{-R} = \frac{1}{a^R}$ for any $a \neq 0$

$a^R \cdot a^S = a^{R+S}$

$(a^R)^S = a^{R*S}$

$\frac{a^R}{a^S} = a^{R-S}$

$a^R \cdot b^R = (ab)^R$

$if \quad x^R = a, \quad then \quad x = a^{\frac{1}{R}} = \sqrt[R]{a}$

$if \quad ax^2 + bx + c = 0, \quad then \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ "quadratic formula"

Exponentials and Logarithms

If $b^x = a$, then $x = \log_b(a)$, where $\log_b$ is read as "logarithm to the base *b*".

Logarithms allow solving many equations where the unknown variable *x* appears as an exponent.

The logarithm function has several very useful relationships or "rules". (The first four relationships are true for any fixed base *b*, so we can omit the *b* when listing them.)

1. $\log(a \cdot c) = \log(a) + \log(c)$ [log of a product]
2. $\log\left(\frac{a}{c}\right) = \log(a) - \log(c)$ [log of a fraction]
3. $\log(a^R) = R \cdot \log(a)$ [log of a power]
4. $\log\left(\left(a^R\right)^S\right) = S \cdot R \cdot \log(a)$ [log of power-to-a-power]
5. $\log_c(a) = \log_b(a) / \log_b(c)$ [change of base]
6. if $\log(a) = \log(b)$, then $a = b$